

Effects of Hall and thermal on MHD Stokes' second problem for unsteady second grade fluid flow

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Abstract: In this paper, we investigated the combined effects of Hall and thermal on MHD Stokes' second problem for unsteady second grade fluid flow. The expressions for the velocity field and the temperature field are obtained analytically. The effects of various pertinent parameters on the velocity field and temperature field are studied in detail with the aid of graphs.

Keywords: Hall parameter, MHD, Second grade fluid, Stokes' second problem.

1 INTRODUCTION

With the growing importance of non-Newtonian fluids in modern technology and industries, investigations of such fluids are desirable. A number of industrially important fluids including molten plastics, polymers, pulps, foods and fossil fuels, which may saturate in underground beds are exhibits non-Newtonian behavior. Due to complexity of fluids, several non-Newtonian fluid models have been proposed. In the category of such fluids, second grade fluid is the simplest subclass for which one can hope to gain an analytic solution. Exact analytic solutions for the flows of non-Newtonian fluids are most welcome provided they correspond to physically realistic situations, as they serve a dual purpose. First, they provide a solution to flow that has technical relevance. Second, such solutions can be used as checks against complicated numerical codes that have been developed for much more complex flows. Various studies on the flows of non-Newtonian fluids have been made under different physical aspects. However some recent contributions in the field may be mentioned in Refs.

(Fetecau and Fetecau, 2003; Hayat et al., 2004; Chen et al., 2004; Fetecau and Fetecau, 2005; Tan and Masuoka, 2005).

The motion of a viscous fluid caused by the sinusoidal oscillation of a flat plate is termed as Stokes' second problem by Schlichting (2000). Initially, both the plate and fluid are assumed to be at rest. At time $t = 0+$, the plate suddenly starts oscillating with the velocity $U_0 e^{i\omega t}$. The study of the flow of a viscous fluid over an oscillating plate is not only of fundamental theoretical interest but it also occurs in many applied problems such as acoustic streaming around an oscillating body, an unsteady boundary layer with fluctuations (Tokuda,1968). Penton (1968) have presented a closed-form to the transient component of the solution for the flow of a viscous fluid due to an oscillating plate. Puri and Kythe (1998) have discussed an unsteady flow problem which deals with non-classical heat conduction effects and the structure of waves in Stokes' second problem. Much work has been published on the flow of fluid over an oscillating plate for different constitutive models (Erdogan, 1995; Zeng and Weinbaum,

Past few decades, the study of magnetohydrodynamics flow of electrically conducting fluids in electric and magnetic fields are of considerable interest in modern metallurgical and metal working process. The Hartmann flow is a classical problem that has important applications in MHD power generators and pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, the petroleum industry, purification of crude oil and design of various heat exchangers. For example, Sparrow and Cess (1961) have studied the effect of a magnetic field on the free convection heat transfer from surface. Garandet et al. (1992) have discussed buoyancy driven convection in rectangular enclosure with a transverse magnetic field. Chamkha (1999) have analyzed free convection effects on three-dimensional flow over a vertical stretching surface in the presence of a magnetic field. Erdogan (2000) analyzed the unsteady flow of viscous fluid due to an oscillating plane wall by using Laplace transform technique. Vajravelu and Rivera (2003) discussed the hydromagnetic flow at an oscillating plate. Singh (2003) have studied MHD free convection and mass transfer flow with Hall current, viscous dissipation, joule heating and thermal diffusion. Reddappa et al. (2009) have investigated the non-classical heat conduction effects in Stokes' second problem of a micropolar fluid under the influence of a magnetic field. The pulsatile flow of couple stress fluid through a porous medium with periodic body acceleration and magnetic field was investigated by Rathod and Tanveer (2009).

In view of these, we studied the effects of Hall and thermal in Stokes' second problem for unsteady second grade fluid flow. The expressions for the velocity field and the temperature field are obtained analytically. The effects of various pertinent parameters on the velocity field and temperature field are studied in detail with the aid of graphs.

2 MATHEMATICAL FORMULATION

We consider the one-dimensional unsteady flow of a laminar, incompressible second grade fluid past a vertical flat plate in the yz -plane and occupy the space $x > 0$, with x -axis in the vertical direction. A uniform magnetic field B_0 is applied transverse direction to the flow. It is assumed that the transversely applied magnetic field and magnetic Reynolds number are very small and hence the induced magnetic field is negligible. The plate initially at rest and at constant temperature θ_∞ which is the free stream temperature is moved with a velocity $U_0 e^{i\omega t}$ in its own plane along the z -axis, and its temperature is subjected to a periodic heating of the form $(\theta_w - \theta_\infty) e^{i\omega t}$, where $\theta_w \neq \theta_\infty$ is some constant.

Viscoelastic fluids can be modeled by Rivlin - Ericksen constitutive equation

$$\mathbf{S} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 \quad (2.1)$$

where \mathbf{S} is the Cauchy stress tensor, p is the scalar pressure, μ, α_1 and α_2 are the material constants, customarily known as the coefficients of viscosity, elasticity and cross - viscosity, respectively. These material constants can be determined from viscometric flows for any real fluid. \mathbf{A}_1 and \mathbf{A}_2 are Rivlin-Ericksen tensors and they denote, respectively, the rate of strain and acceleration. \mathbf{A}_1 and \mathbf{A}_2 are defined by

$$\mathbf{A}_1 = \nabla\mathbf{V} + (\nabla\mathbf{V})^T \quad (2.2)$$

and
$$\mathbf{A}_2 = \frac{d\mathbf{A}_1}{dt} + \mathbf{A}_1(\nabla\mathbf{V}) + (\nabla\mathbf{V})^T \mathbf{A}_1 \quad (2.3)$$

where d/dt is the material time derivative, \mathbf{V} is the velocity field and ∇ gradient operator and $()^T$

transpose operator. The viscoelastic fluids when modeled by Rivlin-Ericksen constitutive equation are termed as second-grade fluids. A detailed account of the characteristics of second - grade fluids is well documented by Dunn and Rajagopal (1995). Rajagopal and Gupta (1984) have studied the thermodynamics in the form of dissipative inequality (Clausius –Duhem) and commonly accepted the idea that the specific Helmholtz free energy should be a minimum in equilibrium. From the thermodynamics consideration they assumed

$$\mu \geq 0, \quad \alpha_1 > 0, \quad \alpha_1 + \alpha_2 = 0 \quad (2.4)$$

We seek the velocity field of the form

$$V = (u(x, t), 0, 0) \quad (2.5)$$

For this type of flow, equation of continuity is identically satisfied and the balance of linear momentum reduces to the following differential equation (Fetecau and Zierep, 2001).

$$\rho \frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2} + \alpha_1 \frac{\partial^3 u}{\partial x^2 \partial t} - \frac{\sigma B_0^2}{1+m^2} u + \rho g \beta (\theta - \theta_0) \quad (2.6)$$

where ρ is the density of the fluid, m is the Hall parameter, g is the acceleration due to gravity, β is the coefficient of the thermal expansion and σ is the electrical conductivity.

The energy equation (MCF model) is given by (Ibrahim et al., 2006)

$$\tau \theta_{tt} + \theta_t = \frac{\chi}{\rho c_p} \theta_{xx} \quad (2.7)$$

Introducing the following non dimensional variables

$$\bar{x} = \frac{u_0}{\nu} x, \quad \bar{u} = \frac{u}{u_0}, \quad \bar{t} = \frac{u_0^2}{\nu} t, \quad \bar{\theta} = \frac{\theta - \theta_0}{\theta_w - \theta_0}$$

into the Eqs. (2.6) and (2.7), we get

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \alpha \frac{\partial^3 \bar{u}}{\partial \bar{x}^2 \partial \bar{t}} + G \bar{\theta} - \frac{M^2}{1+m^2} \bar{u} \quad (2.8)$$

$$p \lambda \frac{\partial^2 \theta}{\partial t^2} + p \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2} \quad (2.9)$$

where

$$\alpha = \frac{\alpha_1 u_0^2}{\mu \nu}, \quad M^2 = \frac{\sigma B_0^2}{\rho u_0^2} \nu, \quad G = \frac{\nu g \beta (\theta_w - \theta_0)}{u_0^3},$$

$$p = \frac{\nu \rho c_p}{\psi}, \quad \lambda = \frac{\tau u_0^2}{\nu}.$$

The corresponding dimensions are boundary conditions are

$$u(0, t) = e^{i\omega t}, \quad \theta(0, t) = e^{i\omega t}$$

$$u(\infty, t) = 0, \quad \theta(\infty, t) = 0 \quad (2.10)$$

3 SOLUTION

To solve the non-linear system (2.8) and (2.9) using the boundary conditions (2.10), we assume that

$$u(x, t) = U(x) e^{i\omega t}, \quad \theta(x, t) = \Theta(x) e^{i\omega t} \quad (3.1)$$

Substituting Eq. (3.1) into Eqs. (2.8) and (2.9) and the boundary conditions (2.10), we get

$$\frac{d^2 U}{dx^2} - N^2 U = -G n \Theta \quad (3.2)$$

$$\frac{d^2 \Theta}{dx^2} + (\lambda p \omega^2 - i \omega p) \Theta = 0 \quad (3.3)$$

here

$$N^2 = \frac{M_1^2 + \omega^2 \alpha + i \omega (1 - \alpha M_1^2)}{1 + \omega^2 \alpha^2}, \quad M_1^2 = \frac{M^2}{1+m^2} \quad \text{and} \quad n = \frac{1 - i \omega \alpha}{1 + \omega^2 \alpha^2}$$

The boundary conditions are

$$U(0) = 1, \quad \Theta(0) = 1$$

$$U(\infty) = 0, \quad \Theta(\infty) = 0 \quad (3.4)$$

Solving the equations (3.2) - (3.3) using the boundary conditions Eq. (3.4), we obtain

$$U = e^{-Nx} + \frac{Gn}{k^2 - N^2} [e^{-Nx} - e^{-kx}] \quad (3.5)$$

$$\Theta = e^{-kx} \quad (3.6)$$

where

$$k = \sqrt{-\lambda p \omega^2 + i \omega p} = \sqrt{\omega p \left(\frac{\sqrt{\omega^2 \lambda^2 + 1} - \lambda \omega}{2} \right)} + i \sqrt{\omega p \left(\frac{\sqrt{\omega^2 \lambda^2 + 1} + \lambda \omega}{2} \right)}$$

The final expressions of the velocity field and temperature field are given by

$$u = \left(e^{-Nx} + \frac{Gn}{k^2 - N^2} [e^{-Nx} - e^{-kx}] \right) e^{i\omega t} \tag{3.6}$$

$$\Theta = e^{-kx + i\omega t} \tag{3.7}$$

4 DISCUSSION OF THE RESULTS

Figs. 1 - 16 show the effects of various values of the pertinent parameters α , G , M , m , p and λ on the velocity ($\text{Re } u$ and $|u|$) and temperature ($\text{Re } \theta$ and $|\theta|$) profiles.

Fig. 1 shows the effects of material parameter α on $\text{Re } u$ for $M = 1$, $p = 1$, $\omega = 10$, $m = 0.2$, $t = 0.1$, $\lambda = 0.005$ and $G = 5$. It is found that, the $\text{Re } u$ decreases with increasing α . The same trend is observed from Fig. 2 for $|u|$.

Fig. 3 depicts the effects of G on $\text{Re } u$ for $M = 1$, $m = 0.2$, $p = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$ and $\alpha = 0.01$. It is observed that, the $\text{Re } u$ initially increases and then decreases with increasing G .

Effects of G on $|u|$ for $M = 1$, $m = 0.2$, $p = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$ and $\alpha = 0.01$ is depicted in Fig. 4. It is noted that, the $|u|$ increases with an increase in G .

Fig. 5 shows the effects of Hartmann number M on $\text{Re } u$ for $G = 5$, $m = 0.2$, $p = 1$, $\omega = 10$, $t \neq 0.1$, $\lambda = 0.005$ and $\alpha = 0.01$. It is found that, the $\text{Re } u$ first decreases and then increases with increasing M .

Fig. 6 depicts the effects of M on $|u|$ for $G = 5$, $p = 1$, $m = 0.2$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$ and $\alpha = 0.01$. It is observed that, the $|u|$ decreases with an increase in M .

The effects of Hall parameter m on $\text{Re } u$ for $G = 5$, $M = 1$, $p = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$ and $\alpha = 0.01$ is shown in Fig. 7. It is found that, the $\text{Re } u$ first increases and then decreases with increasing m .

Fig. 8 illustrates the effects of m on $|u|$ for $G = 5$, $p = 1$, $M = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$ and $\alpha = 0.01$. It is observed that, the $|u|$ increases with an increase in m .

Effect of p on $\text{Re } u$ for $G = 5$, $M = 1$, $m = 0.2$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$ and $\alpha = 0.01$ is shown in Fig. 9. It is found that, the $\text{Re } u$ first decreases and then increasing with increasing p .

Effect of p on $|u|$ for $G = 5$, $M = 1$, $m = 0.2$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$ and $\alpha = 0.01$ is depicted in Fig. 10. It is noted that, the $|u|$ decreases on increasing p .

Fig. 11 shows the effects of λ on $\text{Re } u$ for $G = 5$, $m = 0.2$, $p = 1$, $\omega = 10$, $t = 0.1$, $M = 1$ and $\alpha = 0.01$. It is observed that, the $\text{Re } u$ first decreases and then increases with increasing λ .

The effects of λ on $|u|$ for $G = 5$, $p = 1$, $m = 0.2$, $\omega = 10$, $t = 0.1$, $M = 1$ and $\alpha = 0.01$ is

shown in Fig. 12. It is observed that, the $\text{Re}u$ decreases with increasing λ .

Fig. 13 shows the effects of λ on $\text{Re}\theta$ for $G = 5$, $p = 1$, $m = 0.2$, $\omega = 10$, $t = 0.1$, $M = 1$ and $\alpha = 0.01$. It is observed that, the $\text{Re}\theta$ first increases and then decreases with increasing λ .

Fig. 14 depicts the effects of λ on $|\theta|$ for $G = 5$, $p = 1$, $\omega = 10$, $m = 0.2$, $t = 0.1$, $M = 1$ and $\alpha = 0.01$. It is noted that, the $|\theta|$ increases with an increase in λ .

Effects of p on $\text{Re}\theta$ for $G = 5$, $\lambda = 0.005$, $\omega = 10$, $m = 0.2$, $t = 0.1$, $M = 1$ and $\alpha = 0.01$ is depicted in Fig. 15. It is found that, the $\text{Re}\theta$ first decreases and then increases with an increase in p .

Fig. 16 shows the effects of p on $|\theta|$ for $G = 5$, $m = 0.2$, $\lambda = 0.005$, $\omega = 10$, $t = 0.1$, $M = 1$ and $\alpha = 0.01$. It is observed that, the $|\theta|$ decreases with increasing p .

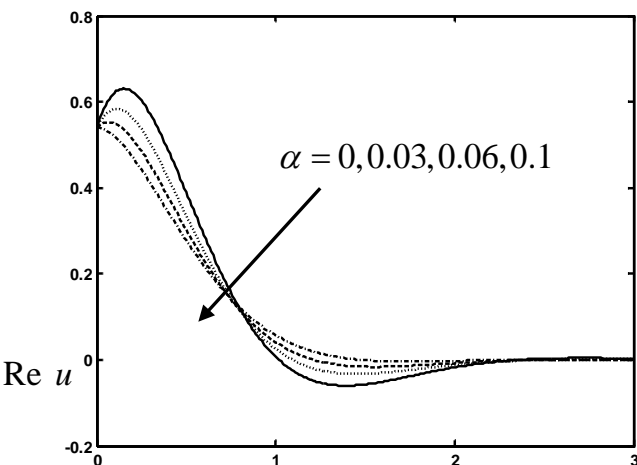


Fig. 1. Effects of α on $\text{Re}u$ for $M = 1$, $p = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$, $m = 0.2$ and $G = 5$

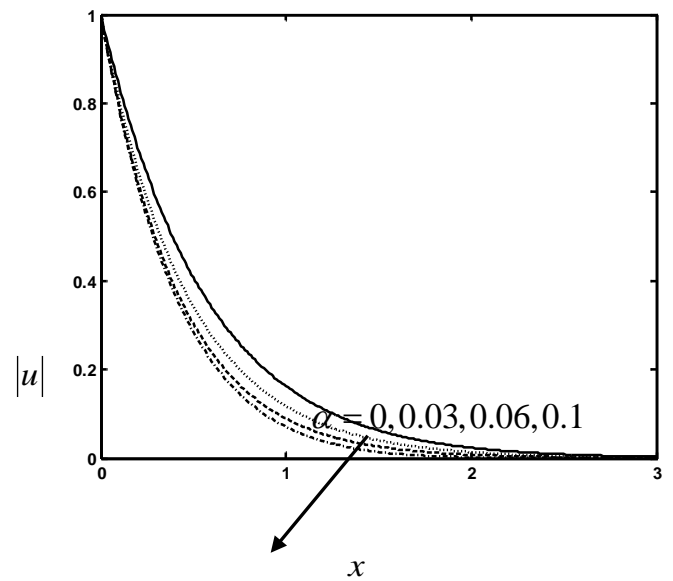


Fig. 2. Effects of α on $|u|$ for $M = 1$, $p = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$, $m = 0.2$ and $G = 5$

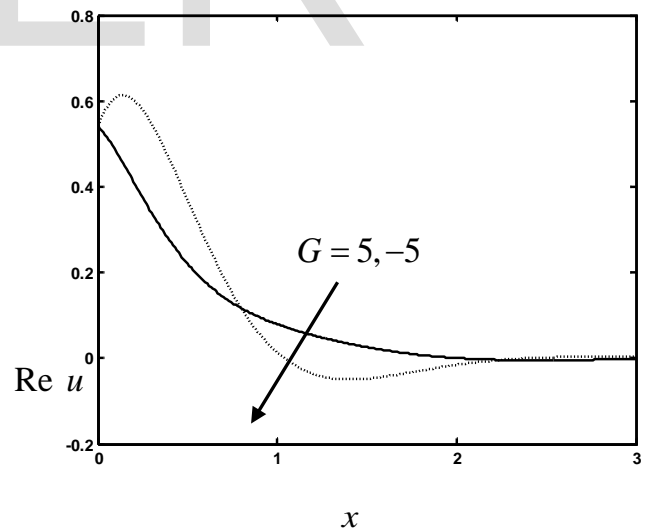


Fig. 3. Effects of G on $\text{Re}u$ for $M = 1$, $p = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$, $m = 0.2$ and $\alpha = 0.01$.

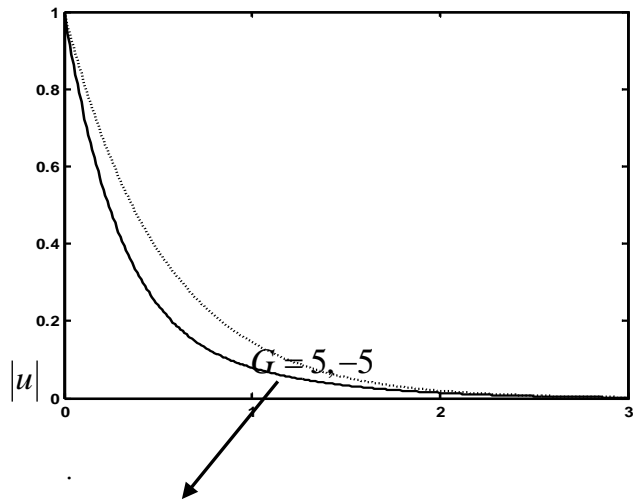


Fig. 4. Effects of G on $|u|$ for $M = 1$, $p = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$, $m = 0.2$ and $\alpha = 0.01$

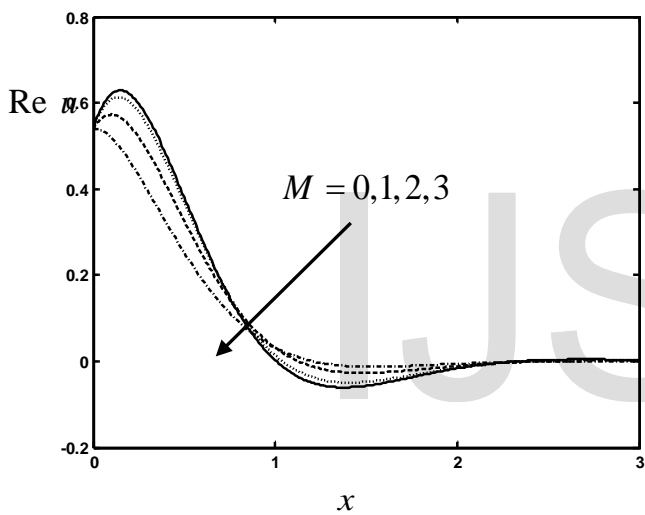


Fig. 5. Effects of M on $\text{Re } u$ for $G = 5$, $p = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$, $m = 0.2$ and $\alpha = 0.01$.

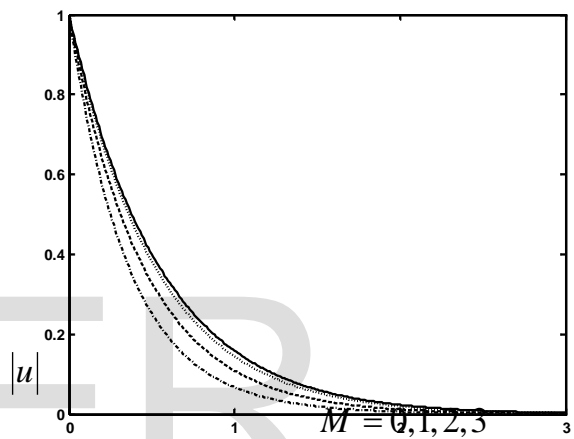


Fig. 6. Effects of M on $|u|$ for $G = 5$, $p = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$, $m = 0.2$ and $\alpha = 0.01$.

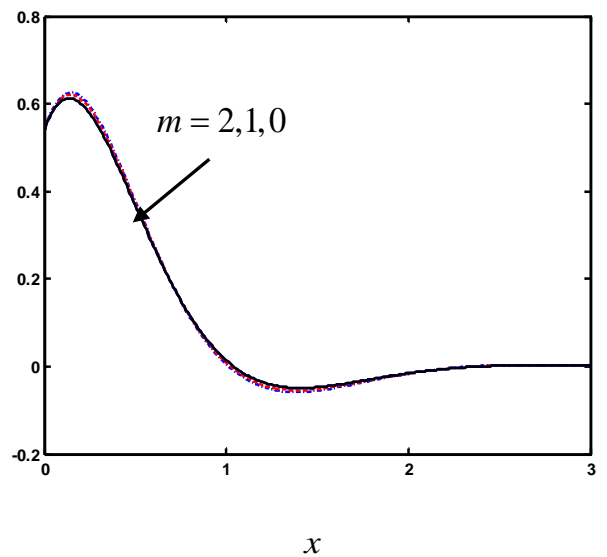


Fig. 7. Effects of m on $\text{Re}u$ for $G = 5$, $p = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$, $M = 1$ and $\alpha = 0.01$.

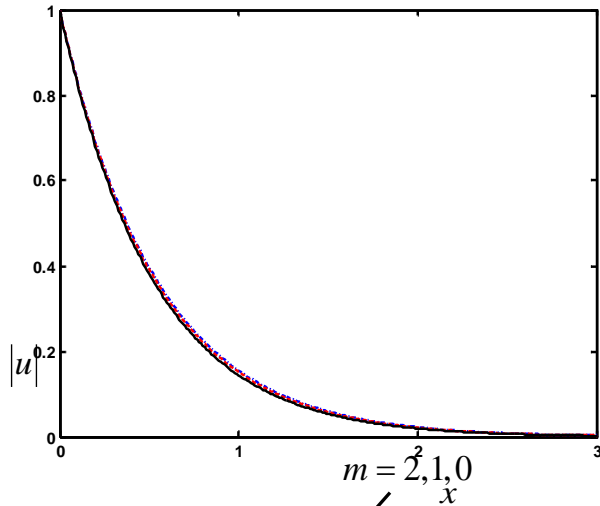


Fig. 8. Effects of m on $|u|$ for $G = 5$, $p = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$, $M = 1$ and $\alpha = 0.01$

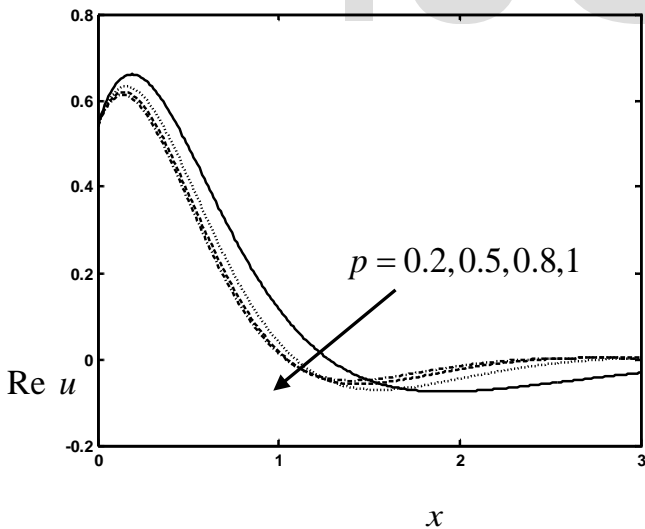


Fig. 9. Effects of p on $\text{Re}u$ for $G = 5$, $M = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$, $m = 0.2$ and $\alpha = 0.01$.

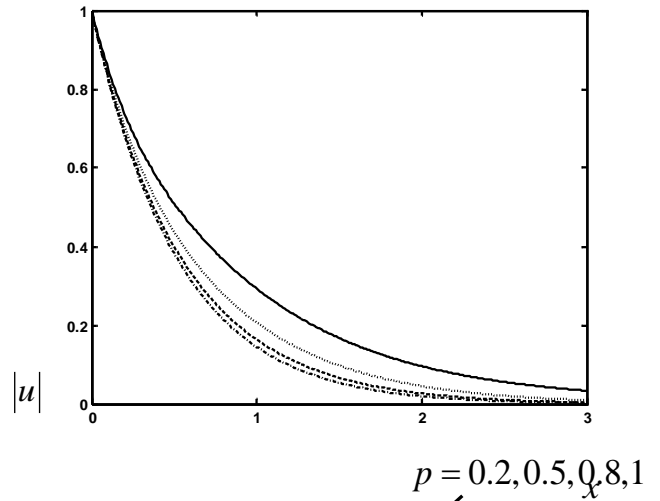


Fig. 10. Effects of p on $|u|$ for $G = 5$, $M = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$, $m = 0.2$ and $\alpha = 0.01$.

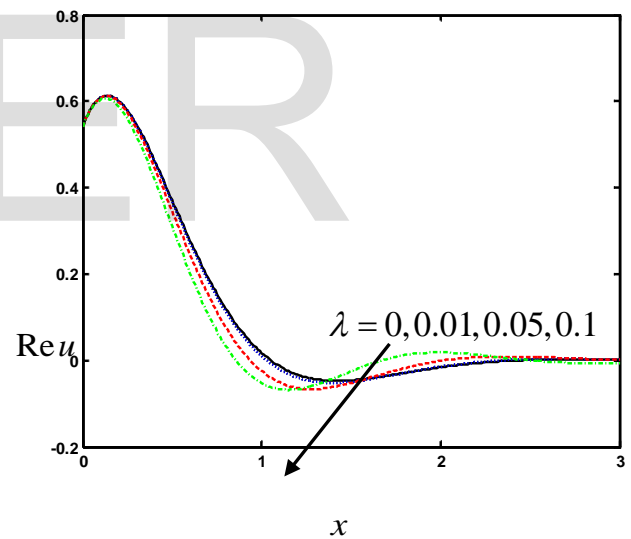


Fig. 11. Effects of λ on $\text{Re}u$ for $G = 5$, $M = 1$, $\omega = 10$, $t = 0.1$, $p = 1$, $m = 0.2$ and $\alpha = 0.01$.

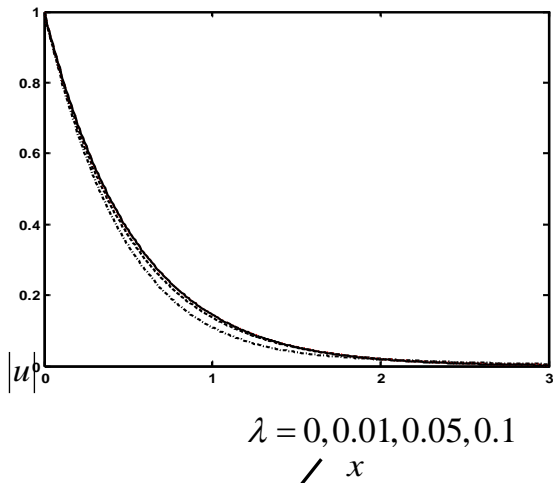


Fig. 12. Effects of λ on $|u|$ for $G = 5$, $M = 1$, $\omega = 10$, $t = 0.1$, $p = 1$, $m = 0.2$ and $\alpha = 0.01$.

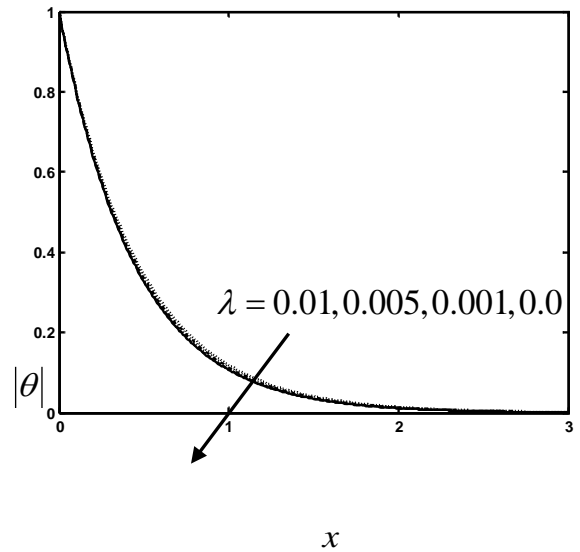


Fig. 14. Effects of λ on $|\theta|$ for $G = 5$, $p = 1$, $\omega = 10$, $t = 0.1$, $m = 0.2$, $M = 1$ and $\alpha = 0.01$.

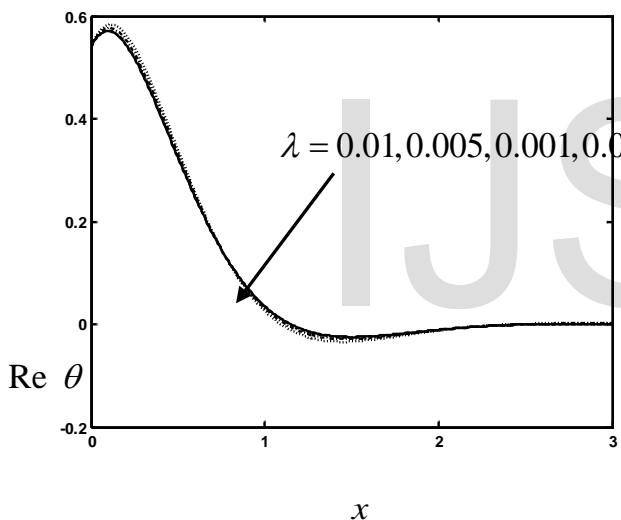


Fig. 13. Effects of λ on $\text{Re} \theta$ for $G = 5$, $p = 1$, $\omega = 10$, $t = 0.1$, $m = 0.2$, $M = 1$ and $\alpha = 0.01$.

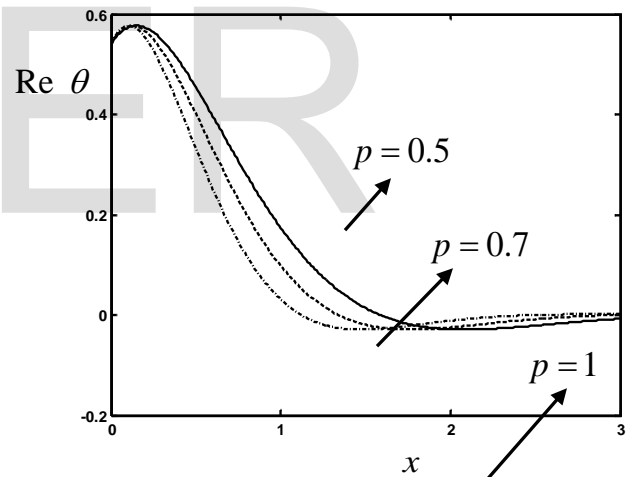


Fig. 15. Effects of p on $\text{Re} \theta$ for $G = 5$, $\lambda = 0.005$, $\omega = 10$, $t = 0.1$, $m = 0.2$, $M = 1$ and $\alpha = 0.01$.

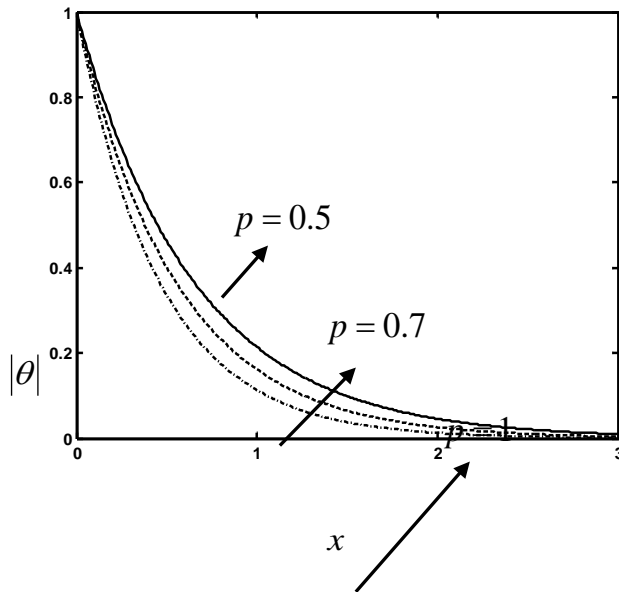


Fig. 16. Effects of p on $|\theta|$ for $G = 5$,
 $\lambda = 0.005$, $\omega = 10$, $t = 0.1$, $m = 0.2$,
 $M = 1$ and $\alpha = 0.01$.

5 CONCLUSIONS

In this paper, the thermal effect in Stokes second problem for unsteady second grade fluid flow under the influence of magnetic field is investigated. The expressions for the velocity field and the temperature field are obtained analytically. It is found that, the $\text{Re } u$ and $|u|$ decreases with increasing α , M , p and λ , while they increases with increasing m and G . The $\text{Re } \theta$ and $|\theta|$ increases with increasing λ , while they decreases with increasing p .

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